**2927★**. [2004 : 172, 174] *Proposed by Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina.*

Suppose that $a$, $b$ and $c$ are positive real numbers. Prove that

$$\frac{a^3}{b^2-bc+c^2}+\frac{b^3}{c^2-ca+a^2}+\frac{c^3}{a^2-ab+b^2} \;\geq\; \frac{3(ab+bc+ca)}{a+b+c}\,.$$

*Comments by Arkady Alt, San Jose, CA, USA.*

The solution of this problem in [2005 : 179–180] is correct; however, an editorial comment at the end of the solution is not correct. It states "Janous supplied a chain of generalizations. First he proved that

$$\sum_{\text{cyclic}}\frac{a^3(b+c)}{b^3+c^3} \;\geq\; 2\sum_{\text{cyclic}}\frac{a^3}{b^2+c^2} \;\geq\; a+b+c\,.$$

He then extended the sharper inequality $\sum\limits_{\text{cyclic}}\frac{a^3}{b^2+c^2}\geq\frac{a+b+c}{2}$ by replacing the left side by $\sum\limits_{\text{cyclic}}\frac{a^{\lambda+1}}{b^\lambda+c^\lambda}$, where $\lambda\geq 0$."

Unfortunately, the inequality

$$\sum_{\text{cyclic}}\frac{a^3\,(b+c)}{b^3+c^3} \;\geq\; 2\sum_{\text{cyclic}}\frac{a^3}{b^2+c^2}$$

is not correct; in fact, the reverse inequality holds. Therefore, the inequality $2\sum\limits_{\text{cyclic}}\frac{a^3}{b^2+c^2} \;\geq\; a+b+c$ is not sharper than $\sum\limits_{\text{cyclic}}\frac{a^3(b+c)}{b^3+c^3} \;\geq\; a+b+c\,.$ Indeed,

$$\frac{a^3\,(b+c)}{b^3+c^3} \;=\; \frac{a^3}{b^2-bc+c^2} \;\leq\; \frac{a^3}{b^2-\frac{1}{2}(b^2+c^2)+c^2} \;=\; \frac{2a^3}{b^2+c^2}\,,$$

so that

$$\sum_{\text{cyclic}}\frac{a^3\,(b+c)}{b^3+c^3} \;\leq\; 2\sum_{\text{cyclic}}\frac{a^3}{b^2+c^2}\,,$$

as claimed.

[*Ed.:* Janous' solution contained a small error which was overlooked by the editor.]